Stokes’ law in complex liquids and inside cell cytoplasm

Karol Makuch[1,2], Robert Hołyst[1], Tomasz Kalwarczyk[1], Piotr Garstecki[1], John F. Brady[2]

[1] Institute of Physical Chemistry, Polish Academy of Sciences
[2] California Institute of Technology

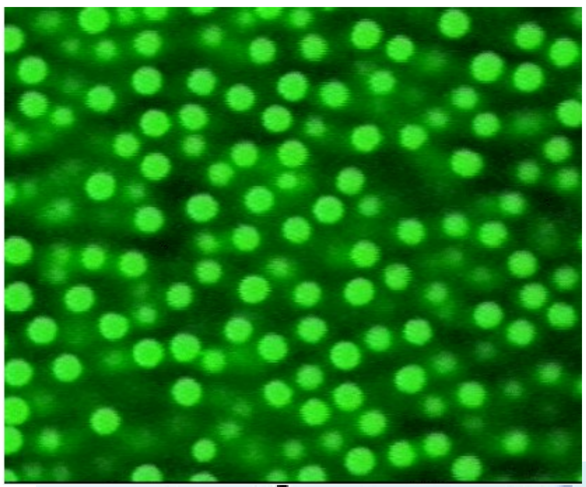
NATIONAL SCIENCE CENTRE POLAND

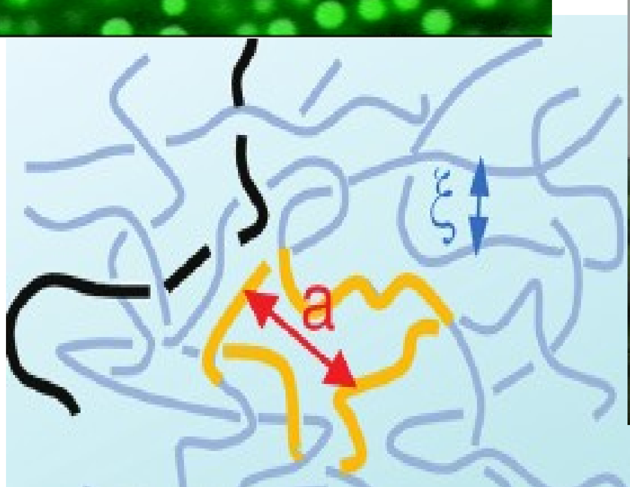
25.11.2019, APS DFD, Seattle

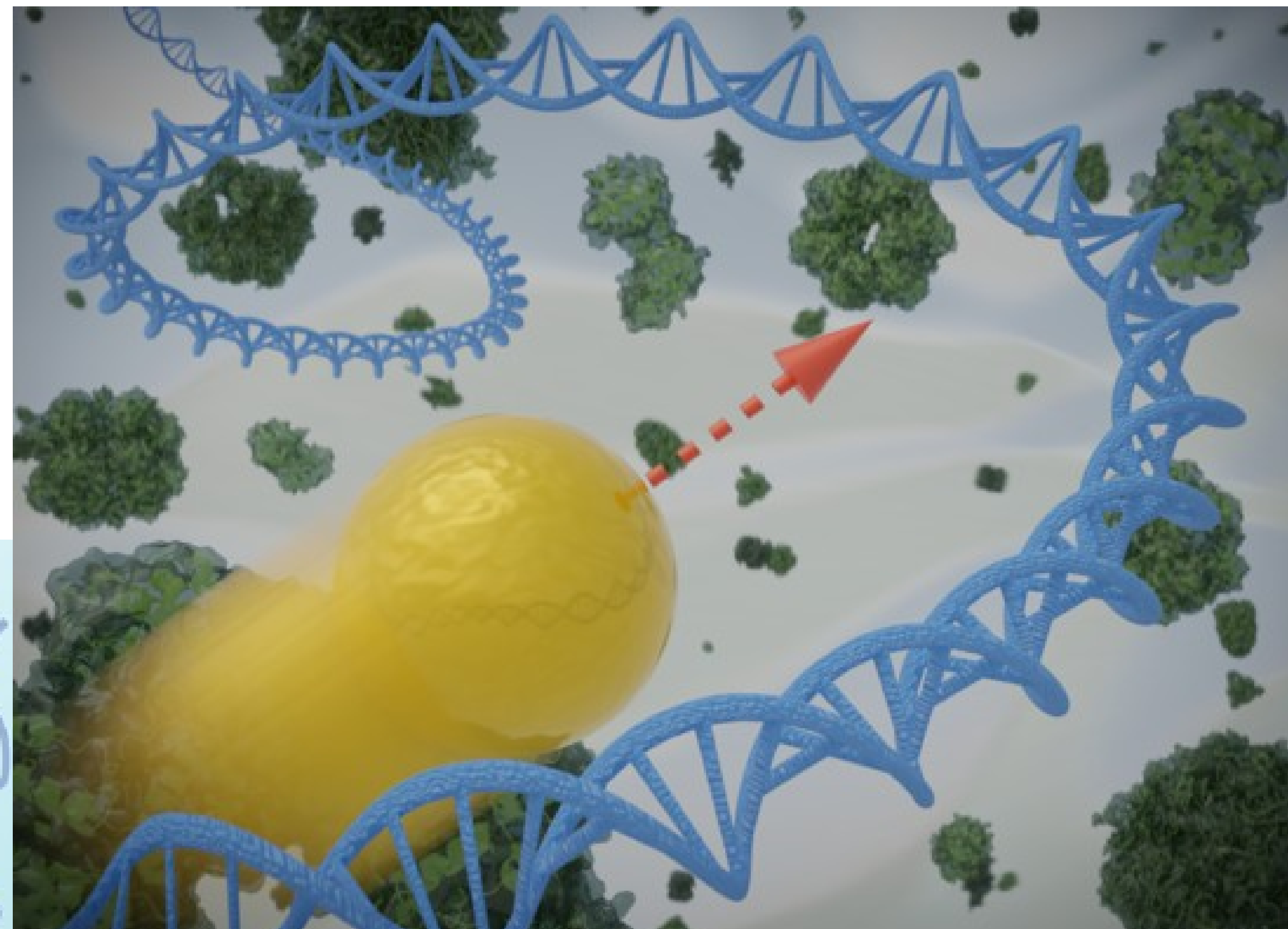
Complex liquid

Liquid consisted of small molecules with macromolecules (>1nm) such as proteins, colloidal spheres, polymers,...

Examples: colloidal suspensions, polymer solutions, cell cytoplasm, ...







Motion of a particle

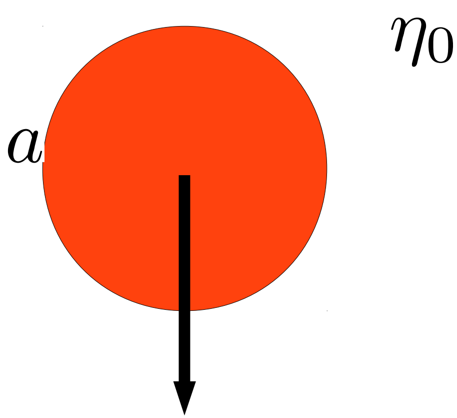
Mechanism of motion – diffusion

Einstein diffusion relation:

$$D = k_B T / \zeta(a)$$

Motion of macromolecules (≤1µm): diffusion

Drag force on a spherical particle
moving slowly in a liquid (Stokes 1851):



$$\mathbf{F} = \zeta(a)\,\mathbf{U}$$

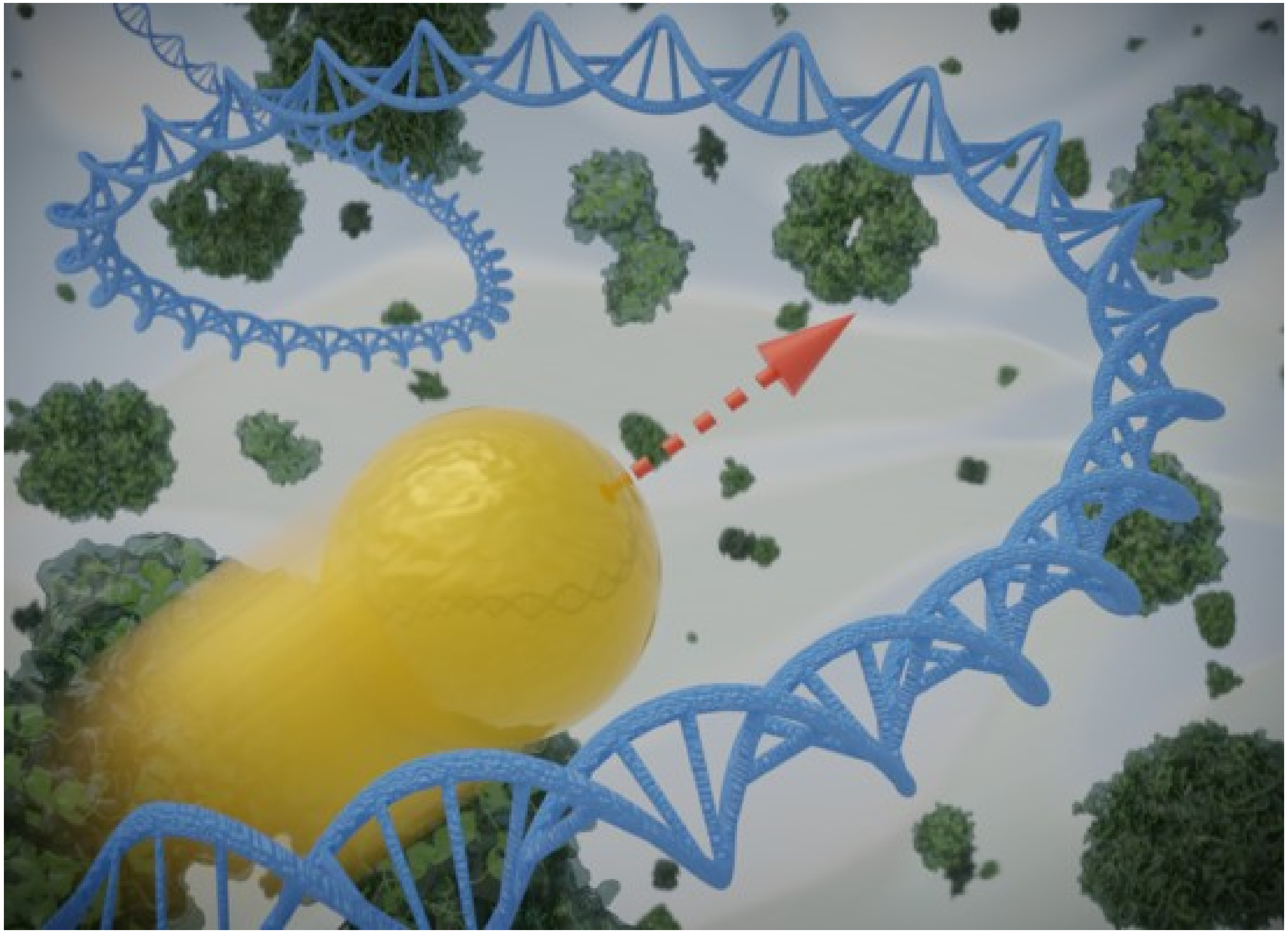
Friction coefficient

Stokes' law in simple liquids:

$$\zeta(a) = 6\pi\eta_0 a$$

Goal: Stokes' law in complex liquids

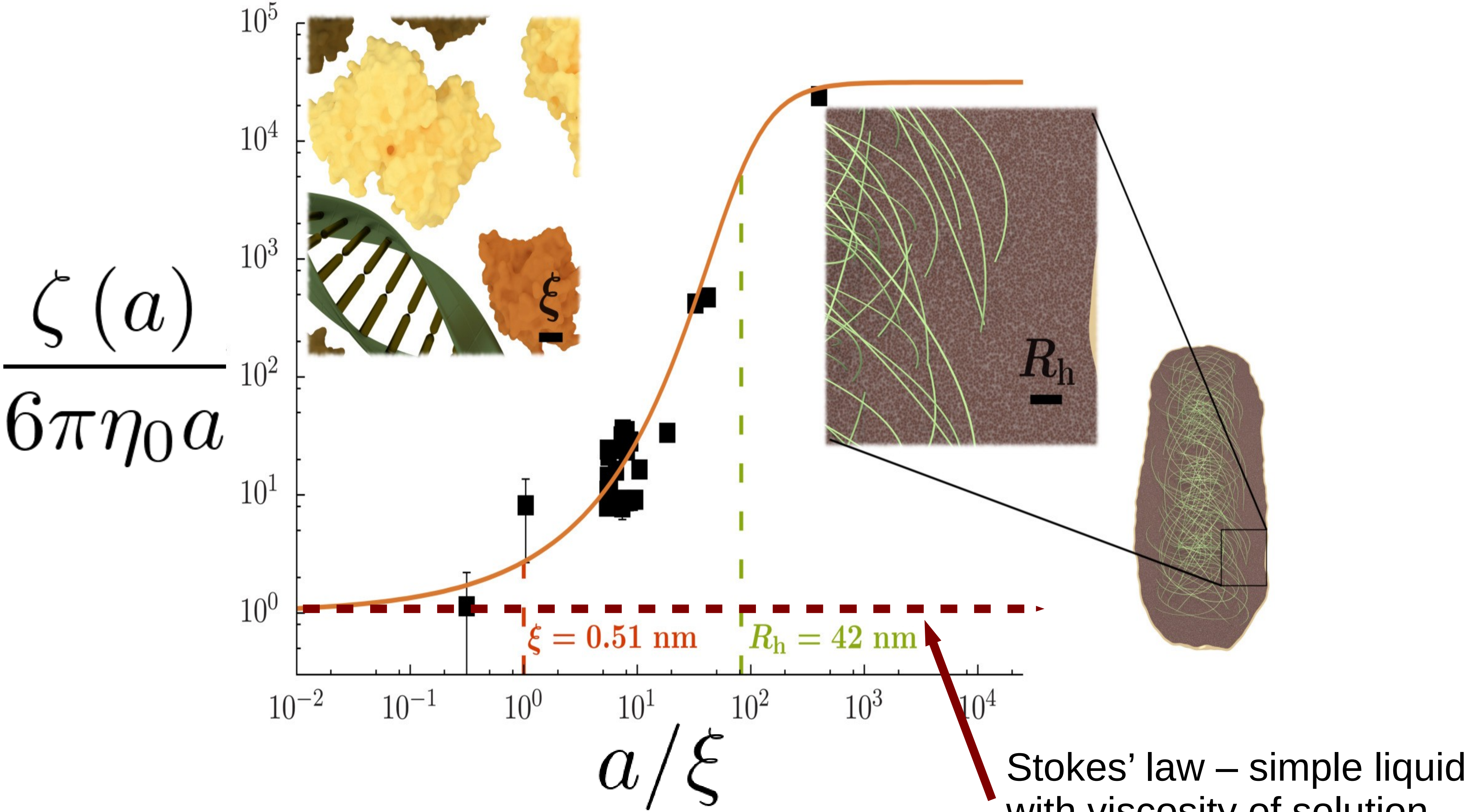
Drag force on a spherical particle moving slowly in complex liquid:



$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient?

Experiments in complex liquids



Diffusion inside *Escherichia coli* cell cytoplasm.

Literature data in [Kalwarczyk et al., Nano Lett. 2011, 11, 2157–2163]

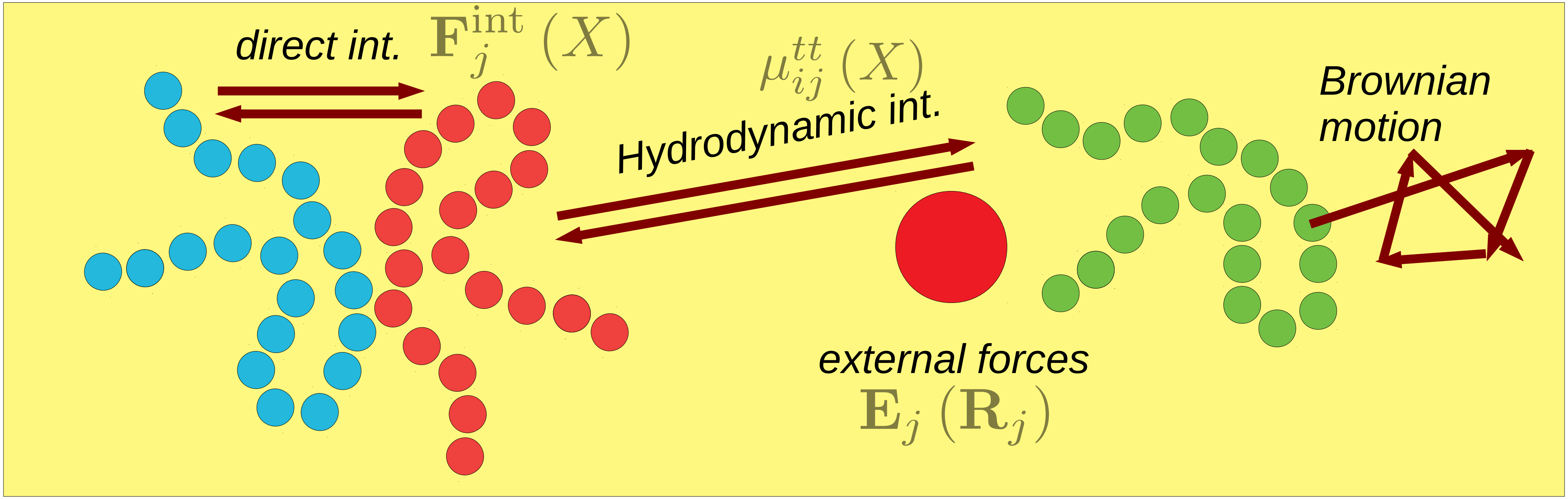
Size of a probe particle is crucial in order to determine its friction

Complex liquid modeled by Smoluchowski dynamics

$$\frac{\partial}{\partial t} P(X,t) =$$

$$= \sum_{i,j=1}^{N} \frac{\partial}{\partial \mathbf{R}_i} \cdot \mu_{ij}^{tt}(X) \cdot \left[k_B T \frac{\partial}{\partial \mathbf{R}_j} - \mathbf{F}_j^{\text{int}}(X) - \mathbf{E}_j(\mathbf{R}_j) \right] P(X,t)$$

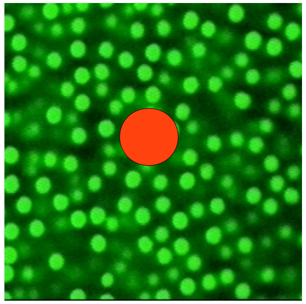
Evolution of probability distr. of interacting beads (macromolecules via bead int.)



Average velocity field around the probe particle

Smoluchowski dynamics [Szymczak and Cichocki (2008)]

$$\langle \mathbf{v}(\mathbf{r}) \rangle = \int d^3r' \; \mathbf{G}_{\text{eff}}(\mathbf{r}-\mathbf{r}')\mathbf{t}^{\text{irr}}(\mathbf{r}')\mathbf{F}$$



b.c.

$$\mathbf{v}\left(a\hat{\mathbf{r}}\right) = \mathbf{U}$$

Effective Green function, does not depend on the probe particle:

$$\hat{\mathbf{G}}_{\text{eff}}(\mathbf{k}) = \frac{1}{\eta\left(k\right)k^2}\left(\mathbf{1}-\hat{\mathbf{k}}\hat{\mathbf{k}}\right)$$

Forces induced in complex liquid by the probe particle.
Contains all 'interactions' between complex liquid and the probe particle.

Theory:
supercooled fluids:
Furukawa, Tanaka (2009)
Suspensions: Beenakker (1984)
No experiments found.

Newton's third law:

$$\int d^3r \, \mathbf{t}^{\text{irr}}\left(\mathbf{r}\right) = \mathbf{1}$$

We introduce phenomenological approximation:

$$\mathbf{t}^{\text{irr}}\left(\mathbf{r}\right) \approx \delta\left(\mathbf{r}\right)\mathbf{1}$$

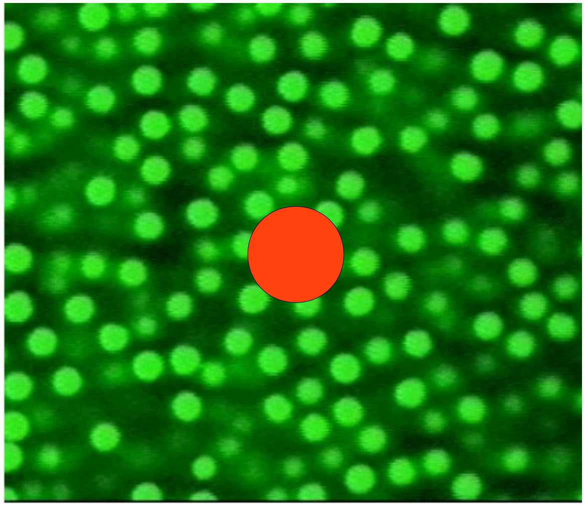
Results: Stokes law in complex liquids
- from friction coefficient to scale dependent viscosity

Stokes law: from scale dependent viscosity to friction coefficient

$$\zeta\left(a\right)=3\pi^{2}/\left(\int_{0}^{\infty}dk\,j_{0}\left(ka\right)/\eta\left(k\right)\right)$$

$$j_0(x) = sin(x)/x$$

Hankel transform

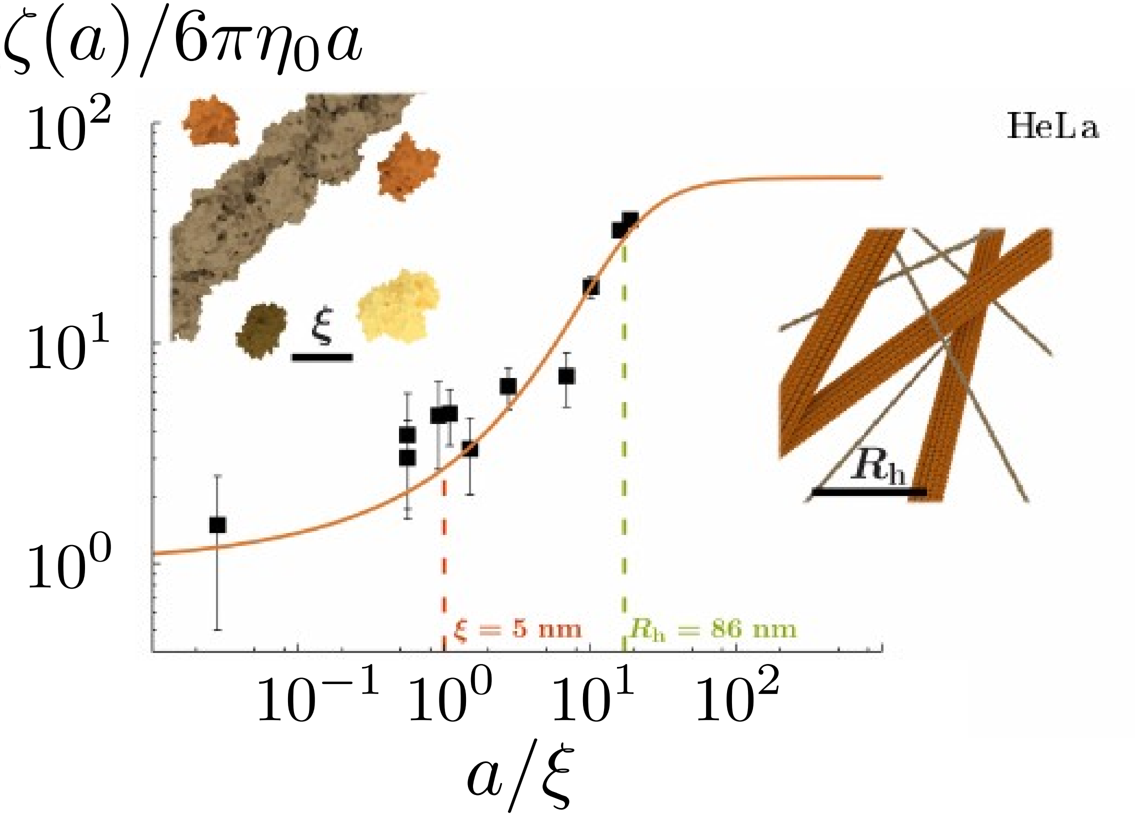


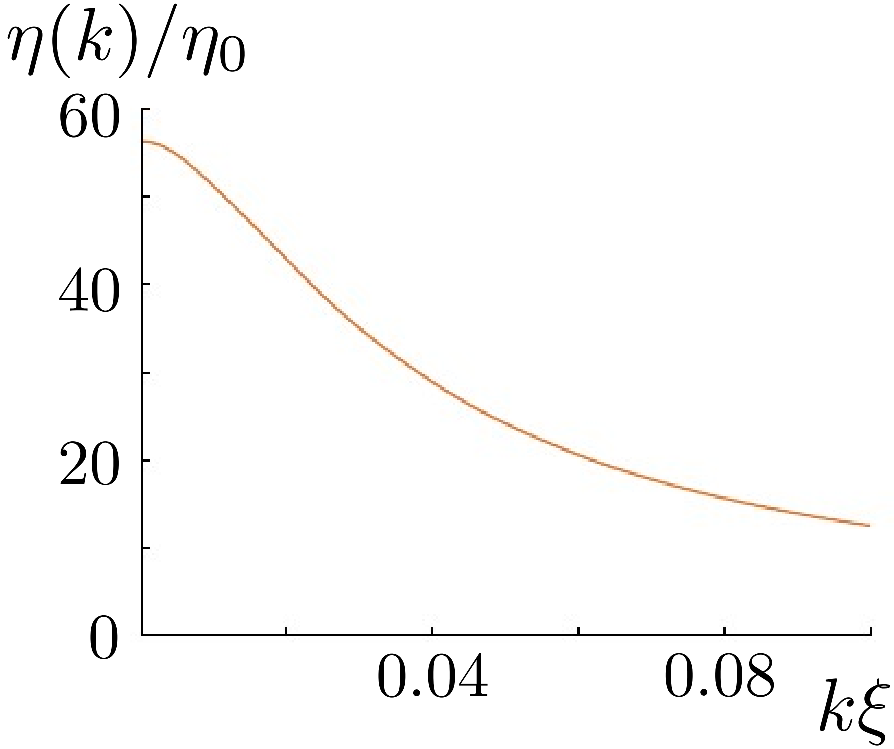
$$\eta\left(k\right)=\frac{1}{6\pi k^{2}}\left[\int_{0}^{\infty}da\,a^{2}\frac{j_{0}\left(ak\right)}{\zeta\left(a\right)}\right]^{-1}$$

Experimental procedure to determine wave-vector dependent viscosity

Application of the Stokes' law in complex liquids - from friction coefficient to scale dependent viscosity

$$\eta(k) = \frac{1}{6\pi k^2}\left[\int_0^\infty da\, a^2 \frac{j_0(ak)}{\zeta(a)}\right]^{-1}$$





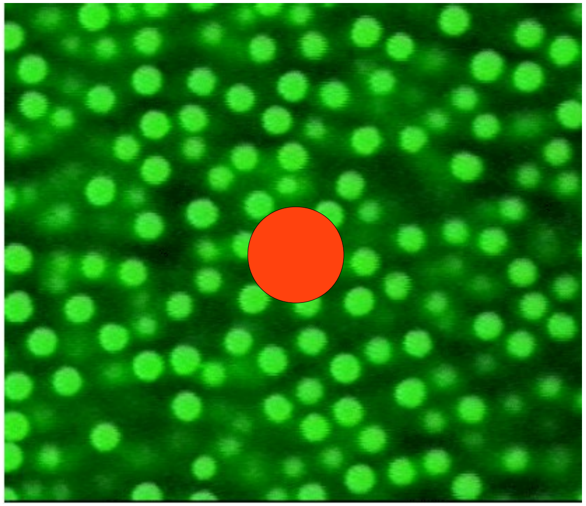
Friction coefficient of different particles inside HeLa cell cytoplasm

Results: Stokes law in complex liquids for rotation, universal formula

$$\mathbf{T} = \zeta_{rot}(a)\mathbf{\Omega}$$

torque

angular velocity



$$\zeta_{\mathrm{rot}}(a) = -4\pi^2 a \left[\frac{d}{da}\int_0^\infty dk \frac{j_0(ak)}{\eta(k)}\right]^{-1}$$

With the Stokes' law for translation we get

$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

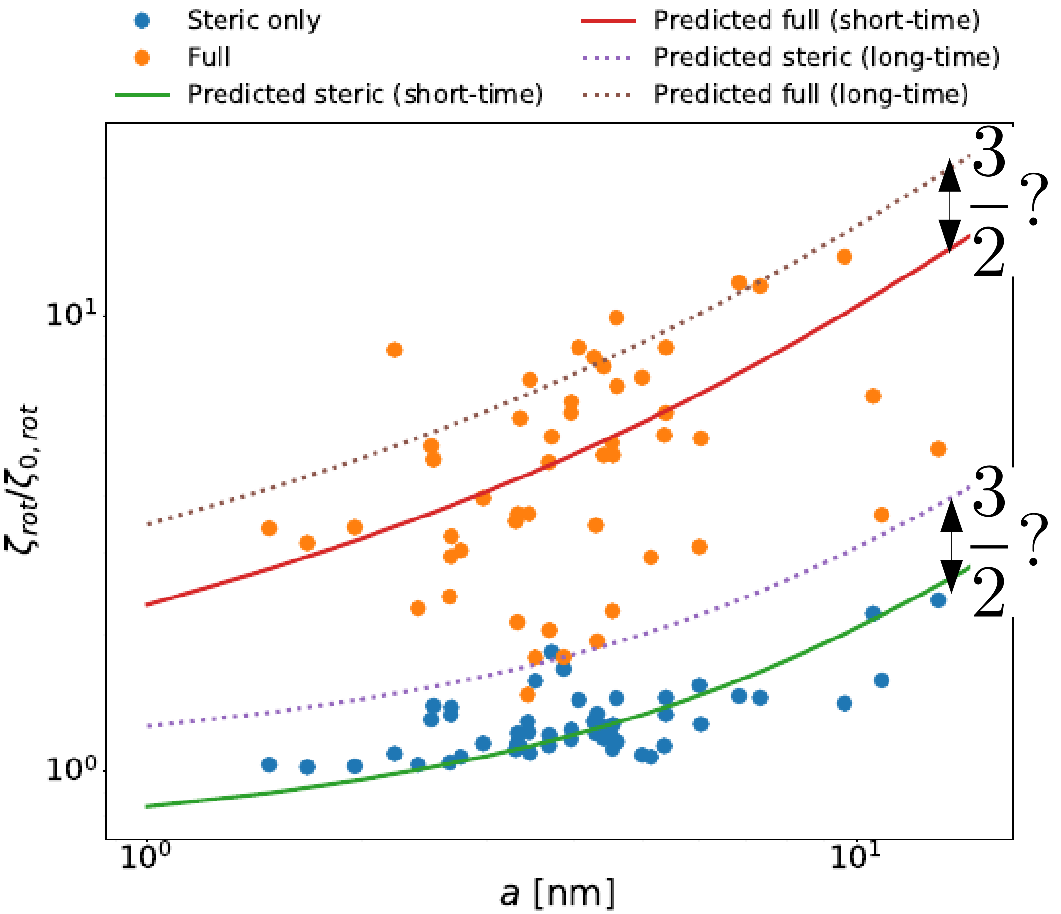
universality

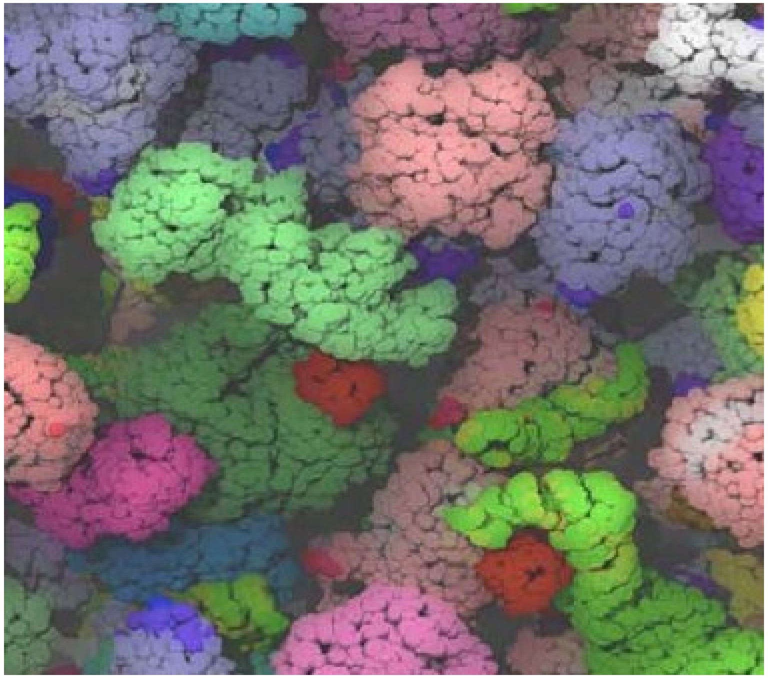
Verification of our phenomenological Stokes' law

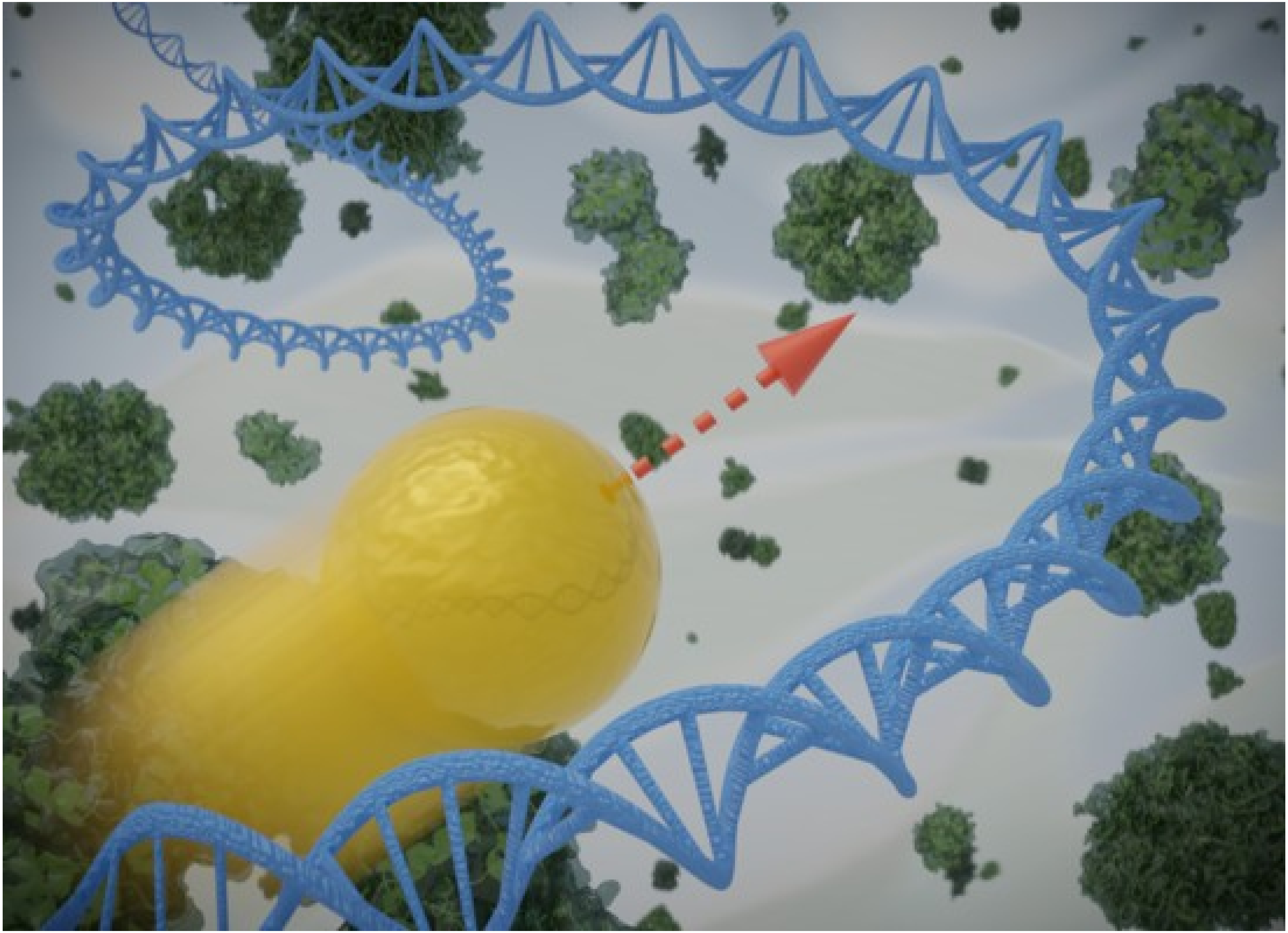
$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

Literature: we didn't find
Experiments: our work, ongoing.
Numerical simulations – similar situation, however...

Sean R. McGuffee, Adrian H. Elcock (2010):







`*Diffusion and flow in complex liquids'* K. Makuch, R. Holyst, T. Kalwarczyk, P. Garstecki, J. F. Brady, Soft Matter, DOI:10.1039/C9SM01119F