Stokes’ law in complex liquids and inside cell cytoplasm

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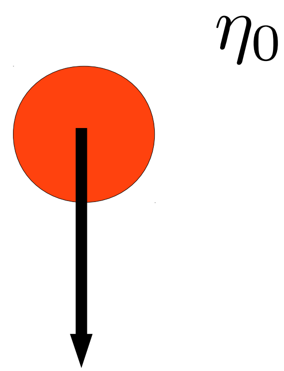
10.10.2019, Cell Physics, Saarbrucken

Stokes' law and friction coefficient

Drag force on a spherical particle
moving slowly in a liquid (Stokes 1851):



$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient

Stokes' law in simple liquids:

$$\zeta\left(a\right) = 6\pi\eta_0 a$$

Einstein diffusion relation:

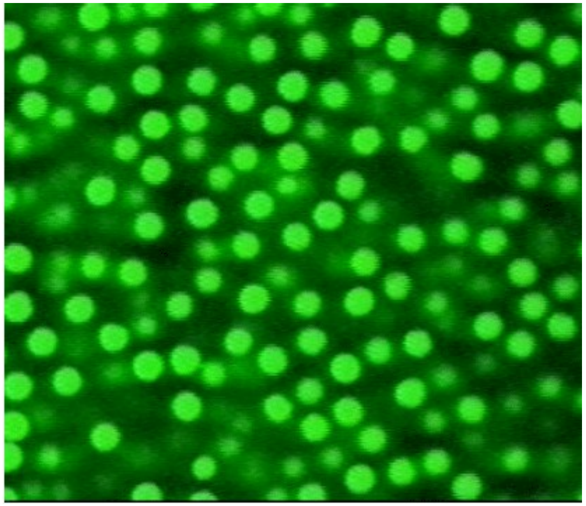
$$D = k_B T/\zeta\left(a\right)$$

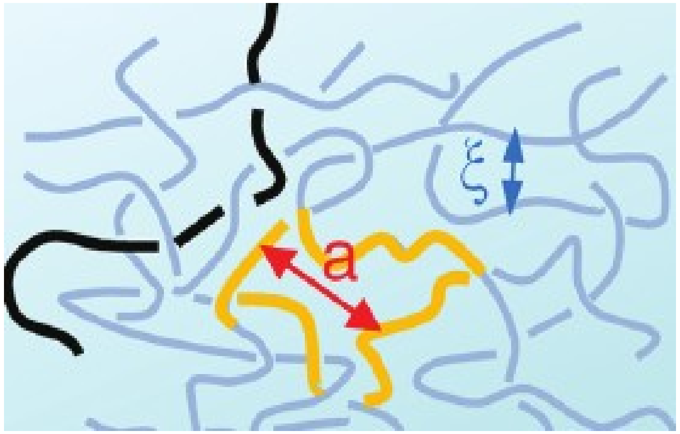
Motion of macromolecules (≤1μm): diffusion

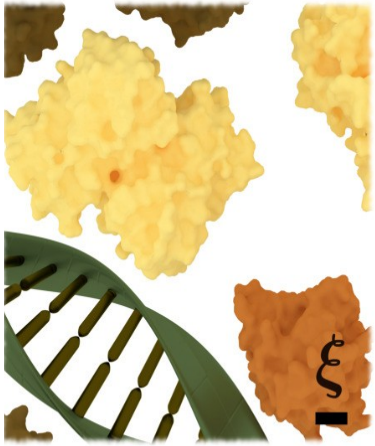
Complex liquid

Liquid consisted of small molecules with macromolecules (>1nm) such as proteins, colloidal spheres, polymers,...

Examples: colloidal suspensions, polymer liquids, cell cytoplasm, ...

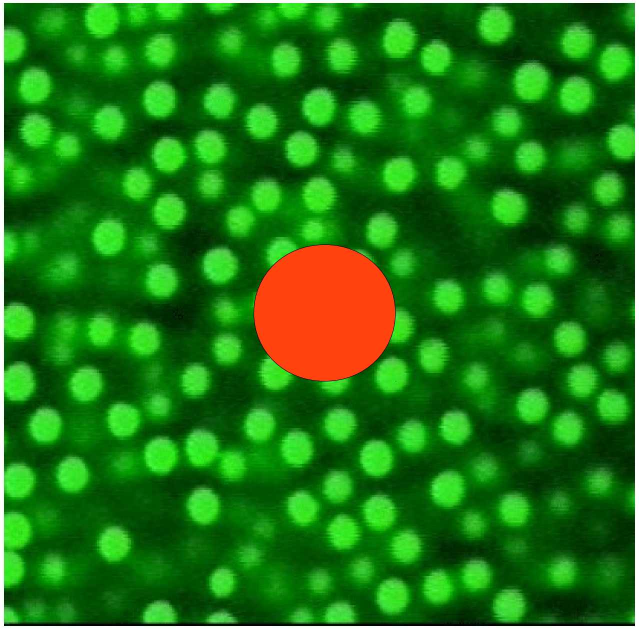






Goal: Stokes' law in complex liquids

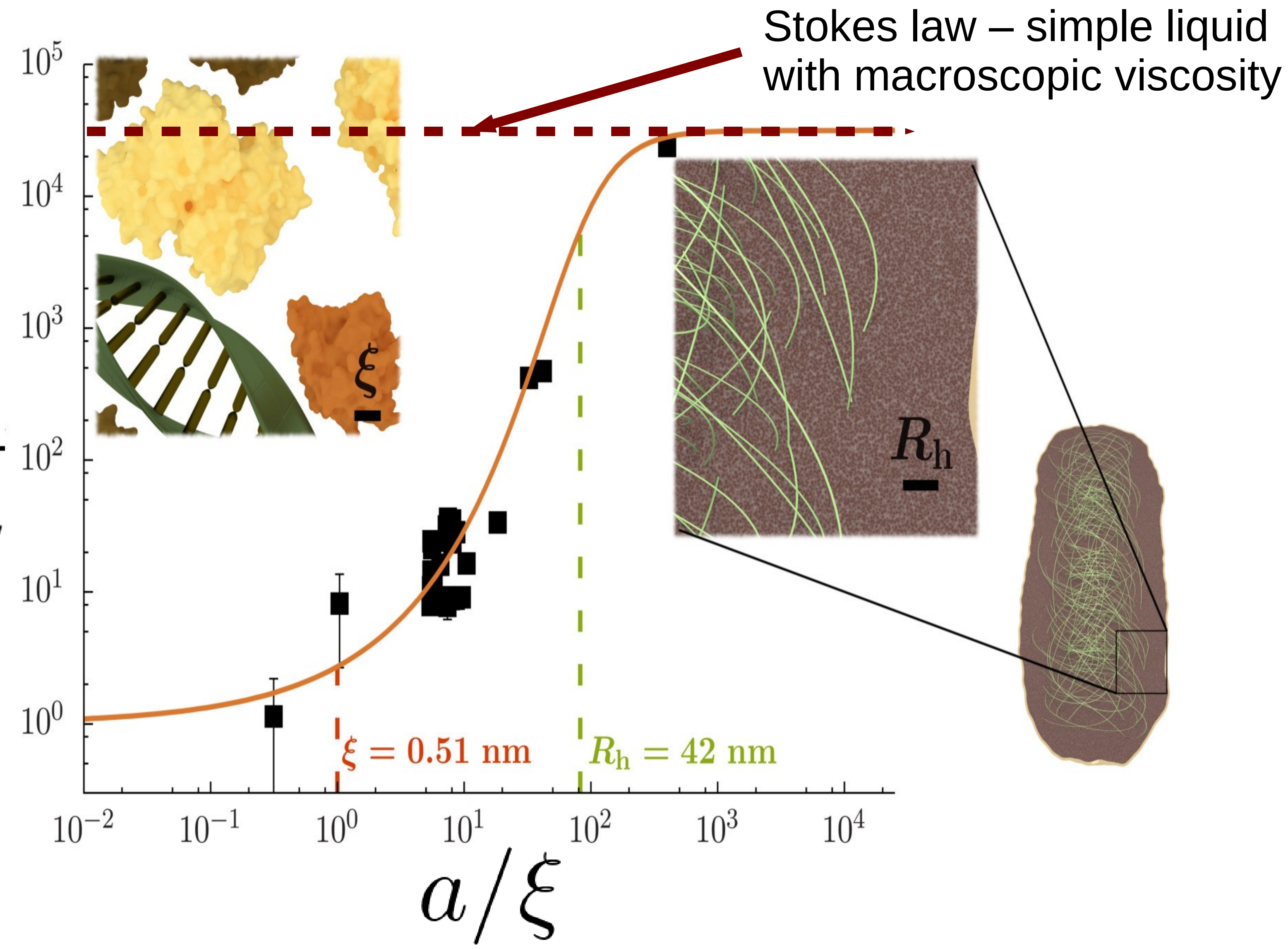
Drag force on a spherical particle moving slowly in complex liquid:



$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient?

Experiments in complex liquids



$$\frac{\zeta(a)}{6\pi\eta_0 a}$$

Diffusion inside *Escherichia coli* cell cytoplasm.

Literature data in [Kalwarczyk et al., Nano Lett. 2011, 11, 2157–2163]

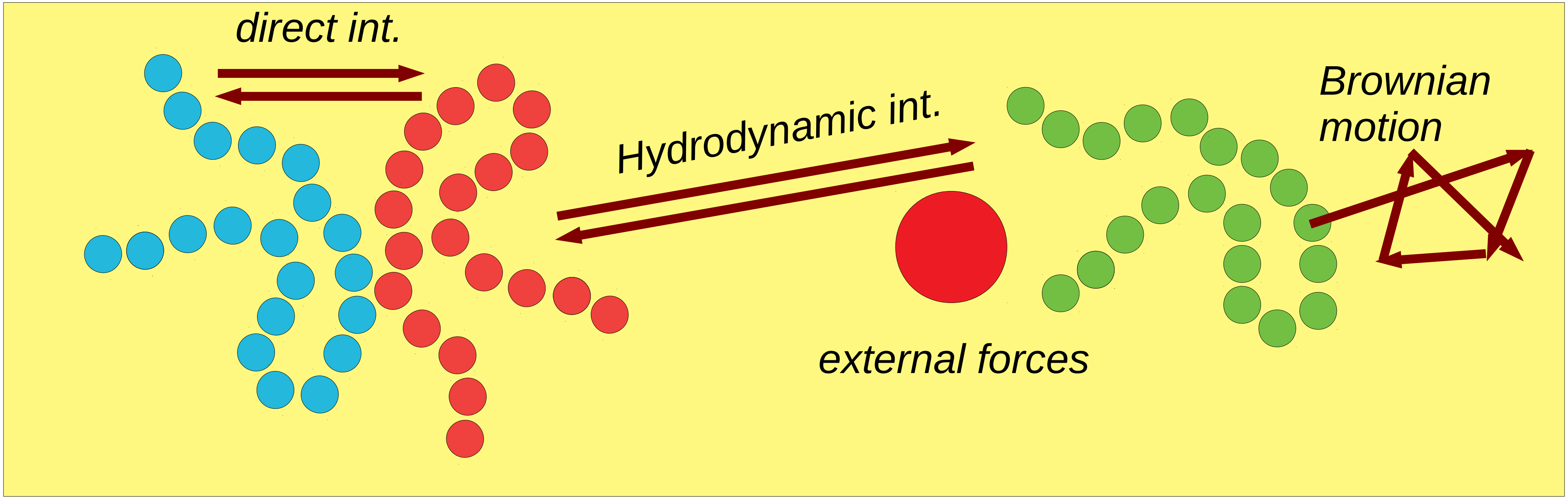
Size of a probe particle is crucial in order to determine its friction

Complex liquid modeled by Smoluchowski dynamics

$$\frac{\partial}{\partial t} P(X,t) =$$

$$= \sum_{i,j=1}^{N} \frac{\partial}{\partial \mathbf{R}_i} \cdot \mu_{ij}^{tt}(X) \cdot \left[k_B T \frac{\partial}{\partial \mathbf{R}_j} + \mathbf{F}_j^{\mathrm{int}}(X) + \mathbf{E}_j(\mathbf{R}_j) \right] P(X,t)$$

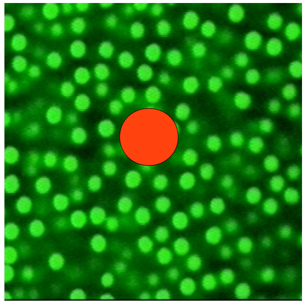
Evolution of probability distr. of interacting beads (macromolecules via bead int.)



Average velocity field around the probe particle

Smoluchowski dynamics [Szymczak and Cichocki (2008)]

$$\langle \mathbf{v}(\mathbf{r})\rangle = \int d^3r' \; \mathbf{G}_{\mathrm{eff}}(\mathbf{r}-\mathbf{r}')\mathbf{t}^{\mathrm{irr}}(\mathbf{r}')\mathbf{F}$$



b.c.

$$\mathbf{v}\left(a\hat{\mathbf{r}}\right) = \mathbf{U}$$

Effective Green function, does not depend on the probe particle:

$$\hat{\mathbf{G}}_{\mathrm{eff}}(\mathbf{k}) = \frac{1}{\eta\left(k\right)k^2}\left(\mathbf{1}-\hat{\mathbf{k}}\hat{\mathbf{k}}\right)$$

Forces induced in complex liquid by the probe particle.
Contains all ‘interactions’ between complex liquid and the probe particle.

Theory:
supercooled fluids:
Furukawa, Tanaka (2009)
Suspensions: Beenakker (1984)
No experiments found.

Newton’s third law:

$$\int d^3r \, \mathbf{t}^{\mathrm{irr}}\left(\mathbf{r}\right) = \mathbf{1}$$

We introduce phenomenological approximation:

$$\mathbf{t}^{\mathrm{irr}}\left(\mathbf{r}\right) \approx \delta\left(\mathbf{r}\right)\mathbf{1}$$

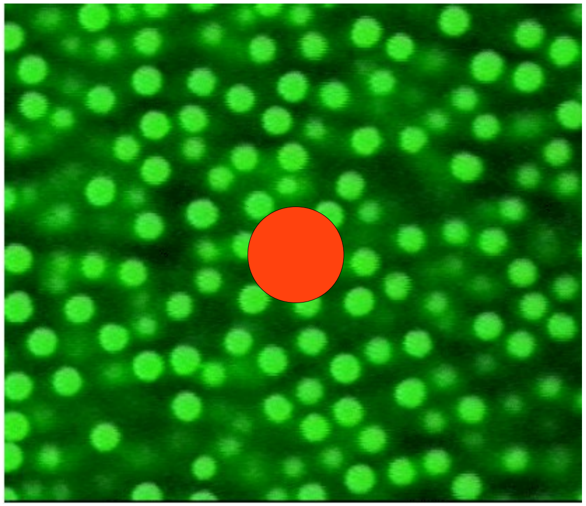
Results: Stokes law in complex liquids
- from friction coefficient to scale dependent viscosity

Stokes law: from scale dependent viscosity to friction coefficient

$$\zeta(a) = 3\pi^2 / \left(\int_0^\infty dk \, j_0(ka) / \eta(k) \right)$$

$$j_0(x) = sin(x)/x$$

Hankel transform



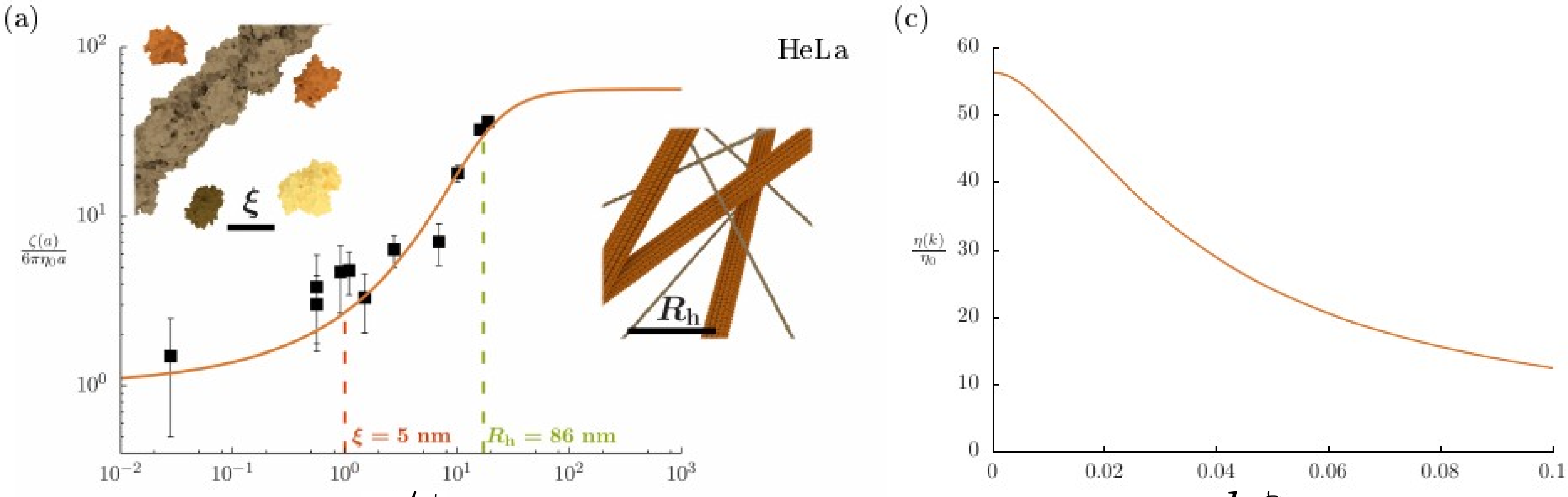
$$\eta(k) = \frac{1}{6\pi k^2} \left[\int_0^\infty da \, a^2 \frac{j_0(ak)}{\zeta(a)} \right]^{-1}$$

Experimental procedure to determine wave-vector dependent viscosity

Application of the Stokes' law in complex liquids
- from friction coefficient to scale dependent viscosity

$$\eta(k) = \frac{1}{6\pi k^2}\left[\int_0^\infty da\, a^2 \frac{j_0(ak)}{\zeta(a)}\right]^{-1}$$

$\zeta(a)/6\pi\eta_0 a$ $\eta(k)/\eta_0$



a/ξ $k\xi$

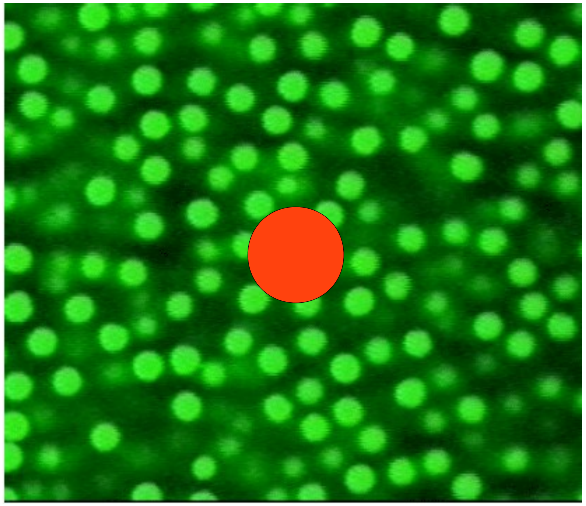
Friction coefficient of different
particles inside HeLa cell cytoplasm

Results: Stokes law in complex liquids for rotation, universal formula

$$\mathbf{T} = \zeta_{rot}(a)\mathbf{\Omega}$$

torque

angular velocity



$$\zeta_{\mathrm{rot}}\left(a\right) = -4\pi^2 a \left[\frac{d}{da}\int_0^\infty dk \frac{j_0\left(ak\right)}{\eta\left(k\right)}\right]^{-1}$$

With the Stokes' law for translation we get

$$\frac{1}{\zeta_{rot}\left(a\right)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta\left(a\right)}$$

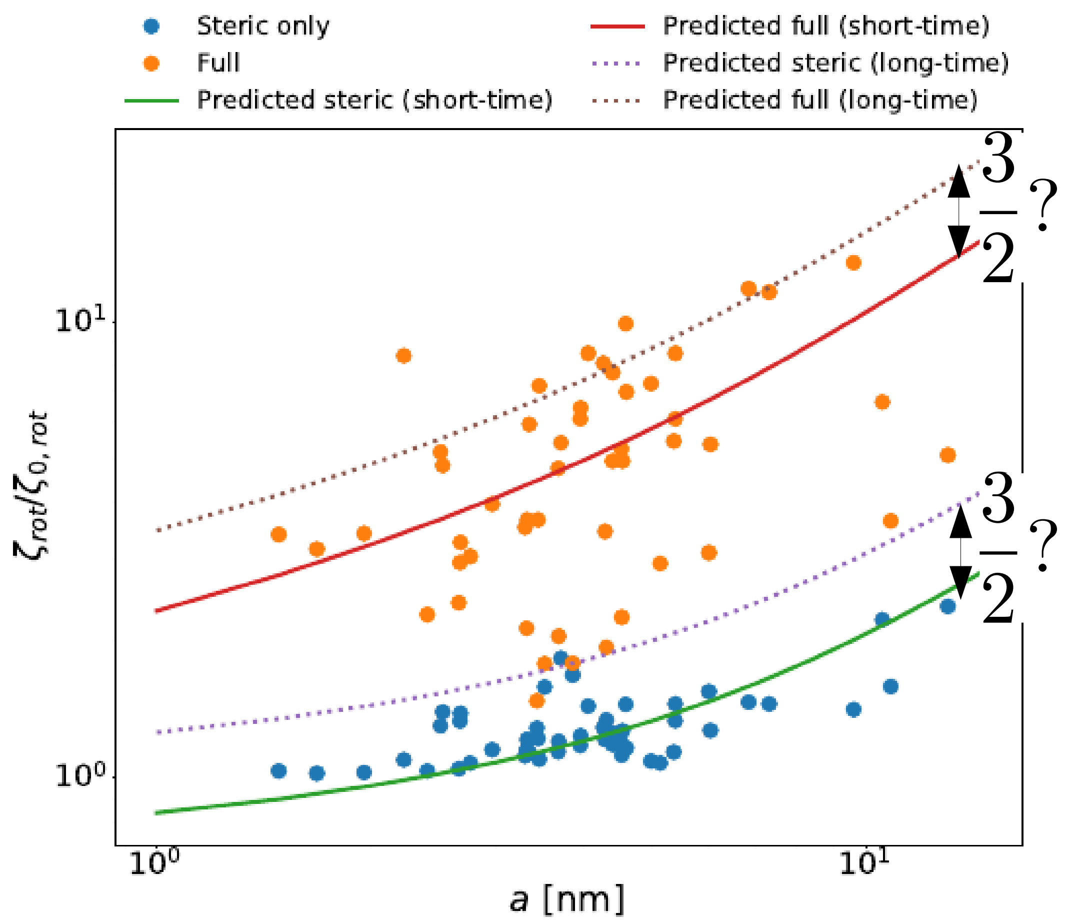
universality

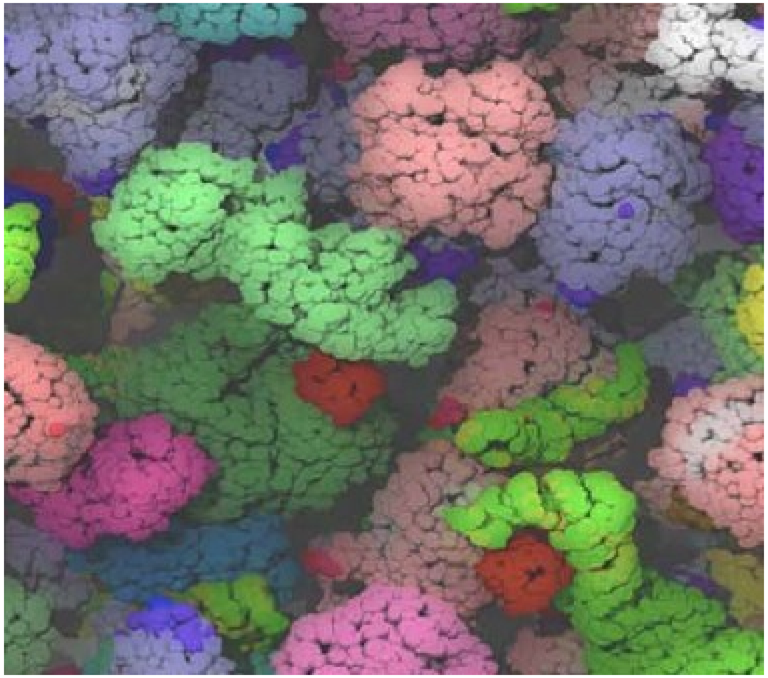
Verification of our phenomenological Stokes' law

$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

Experiments: our work: ongoing, literature: we didn't find
Numerical simulations – similar situation, but...

Sean R. McGuffee, Adrian H. Elcock (2010):





Summary

- Generalization (app) of the Stokes-Einstein formula for complex fluids:

$$\zeta(a) = 6\pi\eta_0 a \longrightarrow \zeta(a) = \frac{3\pi^2}{\int_0^\infty dk\, j_0(ka)/\eta(k)}$$

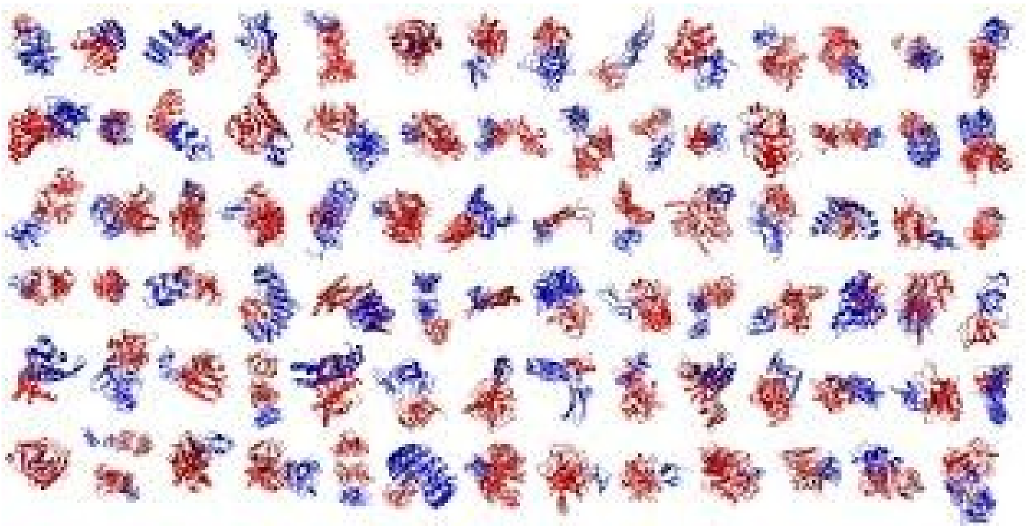
- Experimental procedure to determine viscosity function

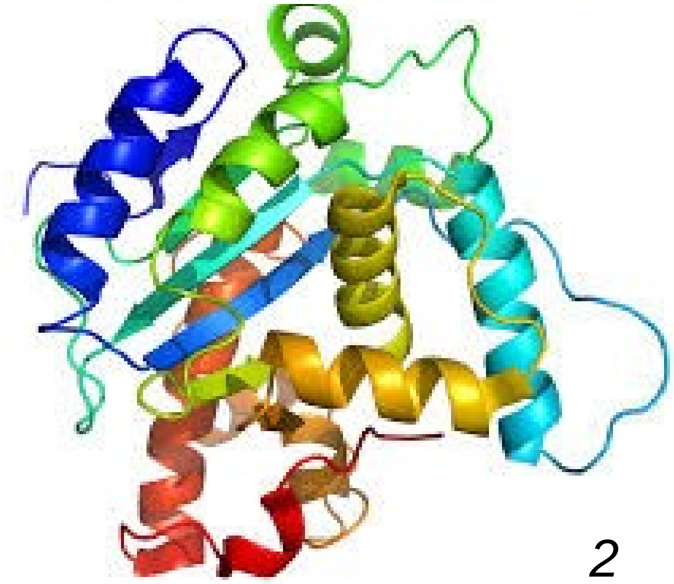
- Universal relation between translational and rotational friction:

$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

Outlook

Simplified description of transport processes in cells...





in $\eta(k)$

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