Stokes’ law in complex liquids and inside cell cytoplasm

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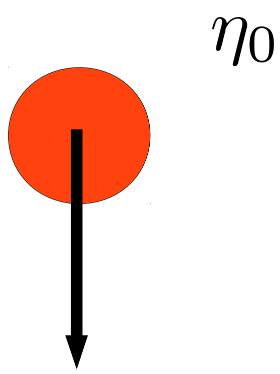
Stokes' law and friction coefficient

Drag force on a spherical particle
moving slowly in a liquid (Stokes 1851):



$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient

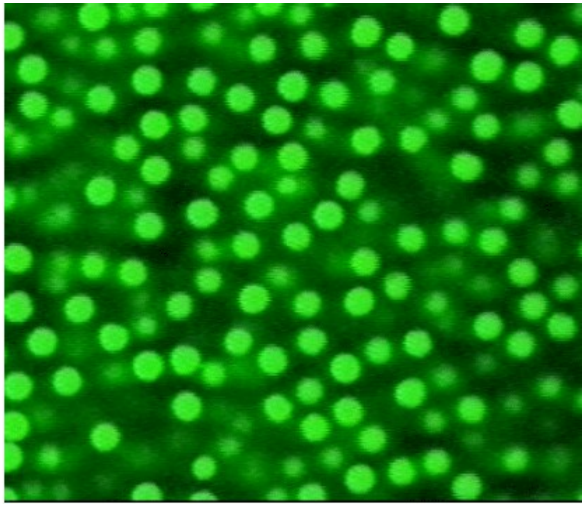
Stokes law in <u>simple liquids</u>:

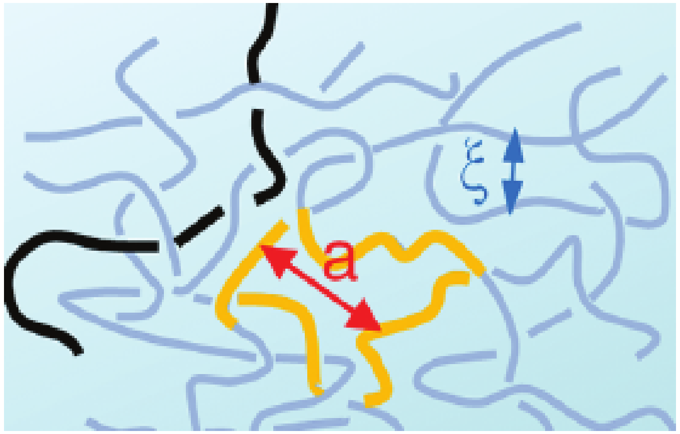
$$\zeta\left(a\right) = 6\pi\eta_0 a$$

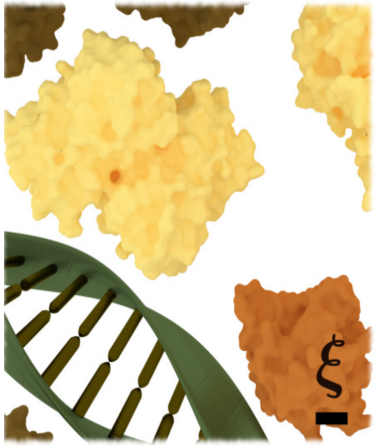
Complex liquid

Liquid consisted of small molecules with macromolecules (>1nm) such as proteins, colloidal spheres, polymers,...

Examples: colloidal suspensions, polymer liquids, cell cytoplasm, ...

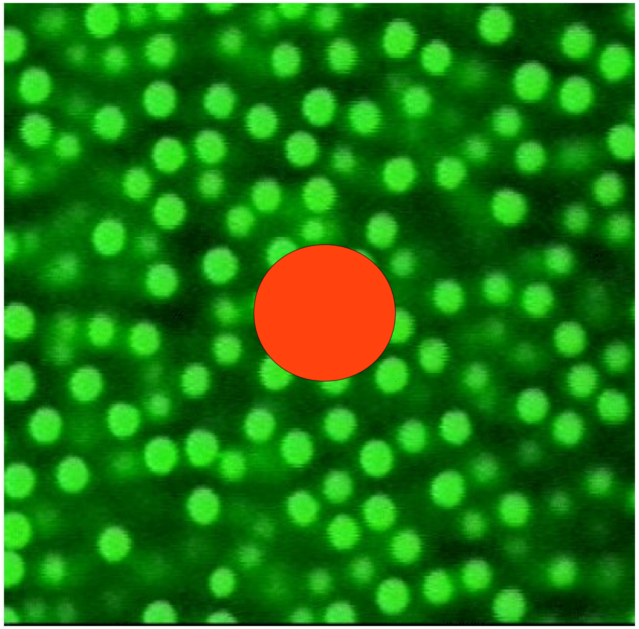






Goal: Stokes' law in complex liquids

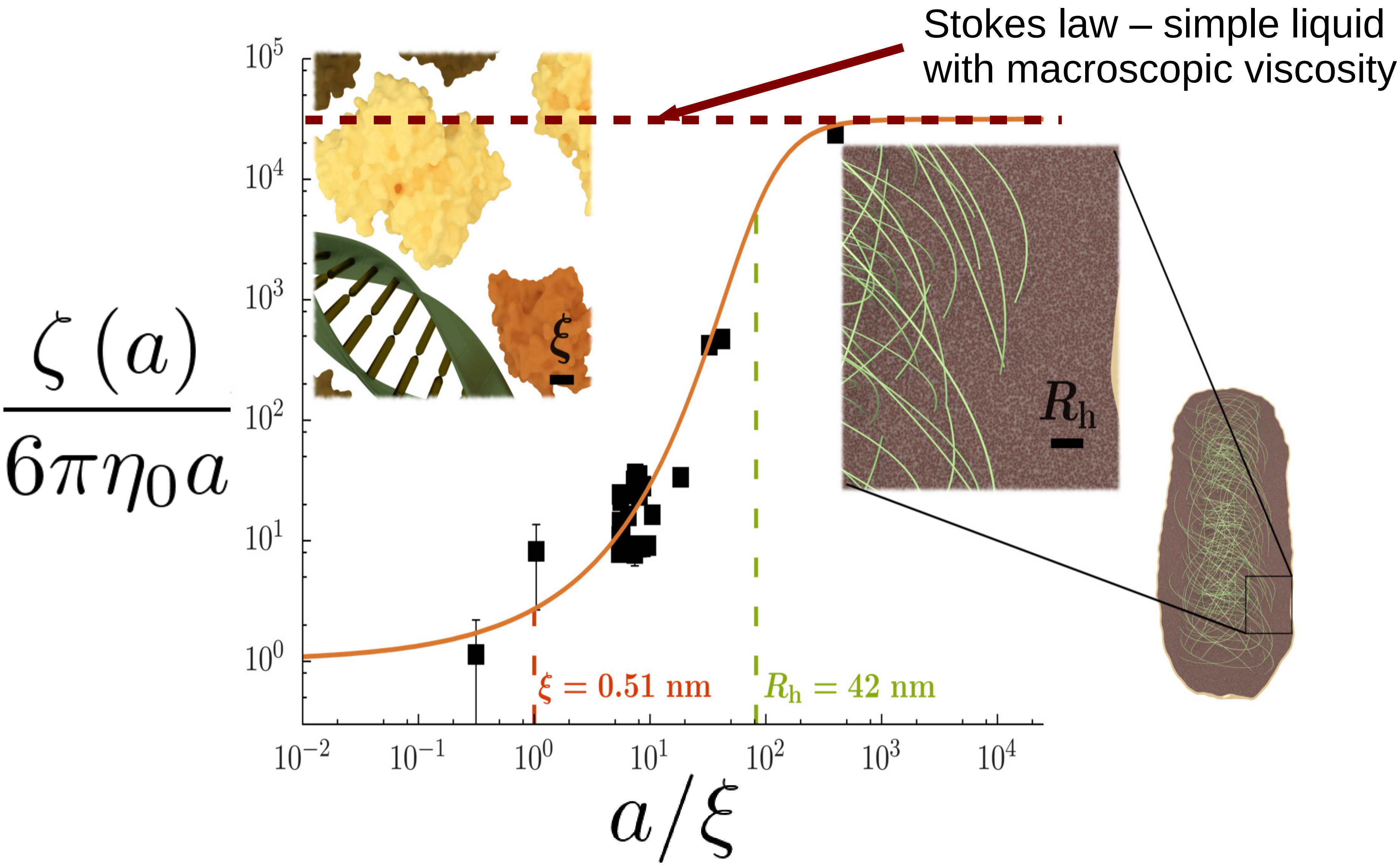
Drag force on a spherical particle
moving slowly in complex liquid:



$$\mathbf{F} = \zeta\left(a\right)\mathbf{U}$$

Friction coefficient

Experiments in complex liquids



Diffusion inside Escherichia Coli cell cytoplasm.

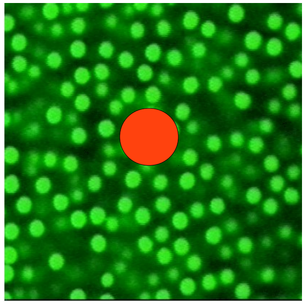
Literature data in [Kalwarczyk et al., Nano Lett. 2011, 11, 2157–2163]

Size of a probe particle is crucial in order to determine the friction

Average velocity field around the probe particle

Smoluchowski dynamics [Szymczak and Cichocki (2008)]

$$\langle \mathbf{v}(\mathbf{r}) \rangle = \int d^3 r' \; \mathbf{G}_{\text{eff}}(\mathbf{r}-\mathbf{r})\mathbf{t}^{\text{irr}}(\mathbf{r}')\mathbf{F}$$



b.c.

$$\mathbf{v}\left(a\hat{\mathbf{r}}\right) = \mathbf{U}$$

Effective Green function, does not depend on the probe particle:

$$\hat{\mathbf{G}}_{\text{eff}}(\mathbf{k}) = \frac{1}{\eta\left(k\right)k^2}\left(\mathbf{1} - \hat{\mathbf{k}}\hat{\mathbf{k}}\right)$$

Forces induced in complex liquid by the probe particle.
Contains all 'interactions' between complex liquid and the probe particle.

Theory:
supercooled fluids:
Furukawa, Tanaka (2009)
Suspensions: Beenakker (1984)
No experiments found.

Newton's third law:

$$\int d^3 r \, \mathbf{t}^{\text{irr}}\left(\mathbf{r}\right) = \mathbf{1}$$

We introduce phenomenological approximation:

$$\boxed{\mathbf{t}^{\text{irr}}\left(\mathbf{r}\right) \approx \delta\left(\mathbf{r}\right)\mathbf{1}}$$

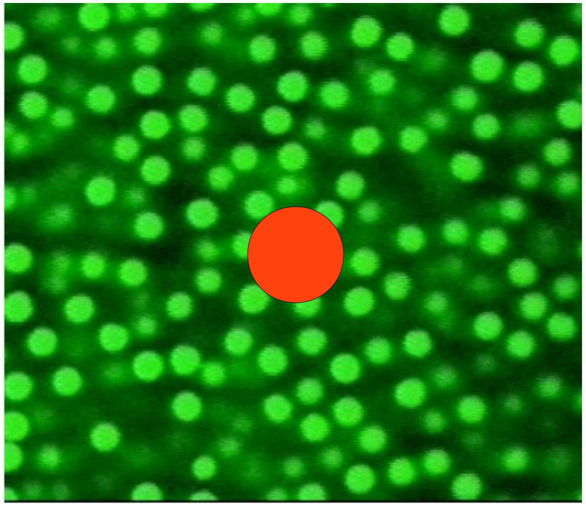
Results: Stokes law in complex liquids
- from friction coefficient to scale dependent viscosity

Stokes law: from scale dependent viscosity to friction coefficient

$$\zeta\left(a\right) = 3\pi^2 / \left(\int_0^\infty dk\, j_0\left(ka\right) / \eta\left(k\right)\right)$$

$$j_0(x) = sin(x)/x$$

Hankel transform

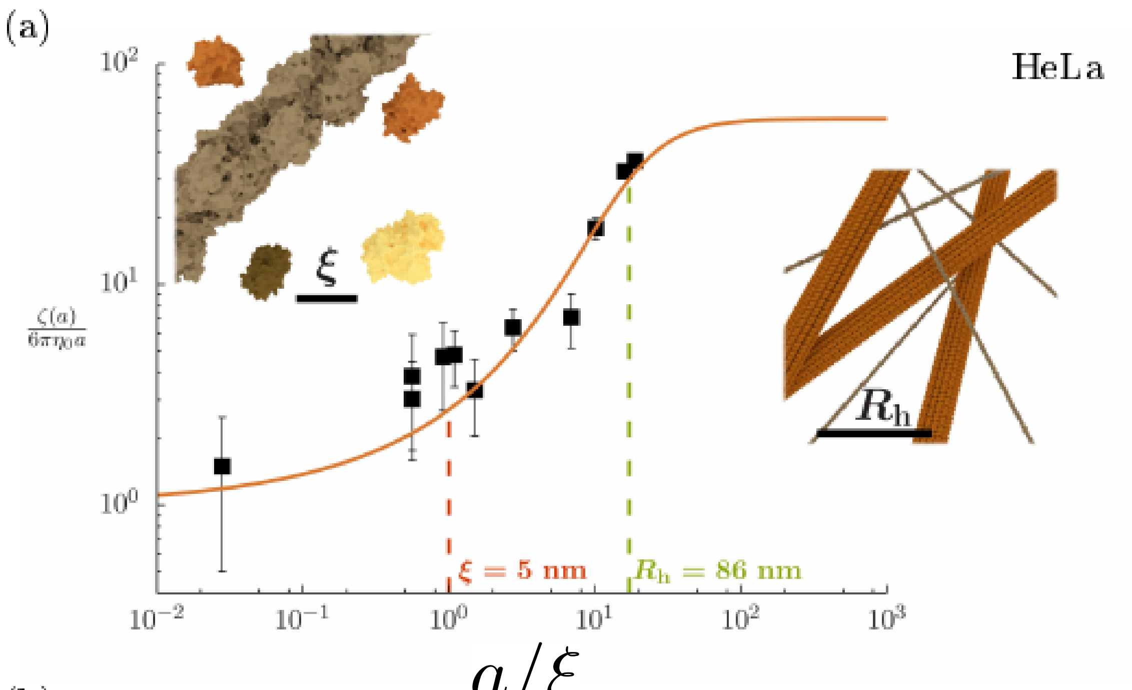


$$\eta\left(k\right) = \frac{1}{6\pi k^2}\left[\int_0^\infty da\, a^2 \frac{j_0\left(ak\right)}{\zeta\left(a\right)}\right]^{-1}$$

Experimental procedure to determine wave-vector dependent viscosity

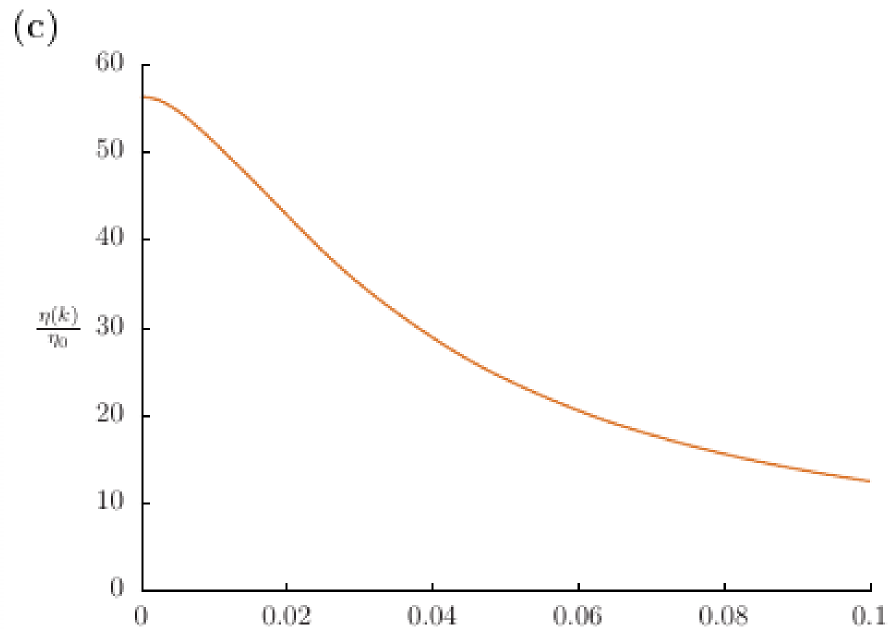
Application of the Stokes' law in complex liquids
- from friction coefficient to scale dependent viscosity

$\zeta(a)/6\pi\eta_0 a$



a/ξ

$\eta(k)/\eta_0$



$k\xi$

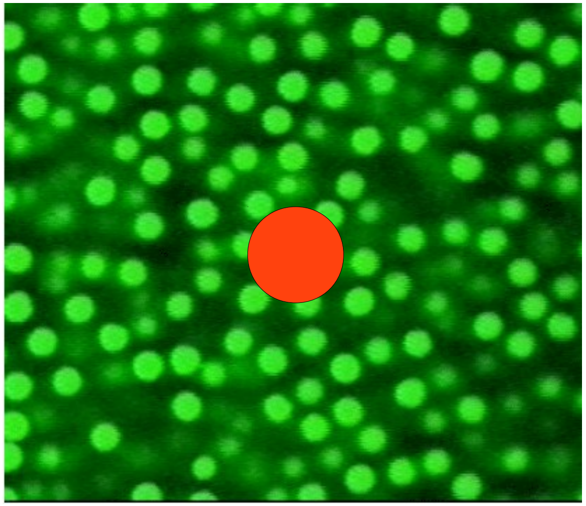
Friction coefficient of different particles inside HeLa cell cytoplasm

Results: Stokes law in complex liquids for rotation, universal formula

$$\mathbf{T} = \zeta_{rot}(a)\mathbf{\Omega}$$

torque

angular velocity



$$\zeta_{\mathrm{rot}}(a) = -4\pi^2 a \left[\frac{d}{da}\int_0^\infty dk \frac{j_0(ak)}{\eta(k)}\right]^{-1}$$

With the Stokes' law for translation we get

$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

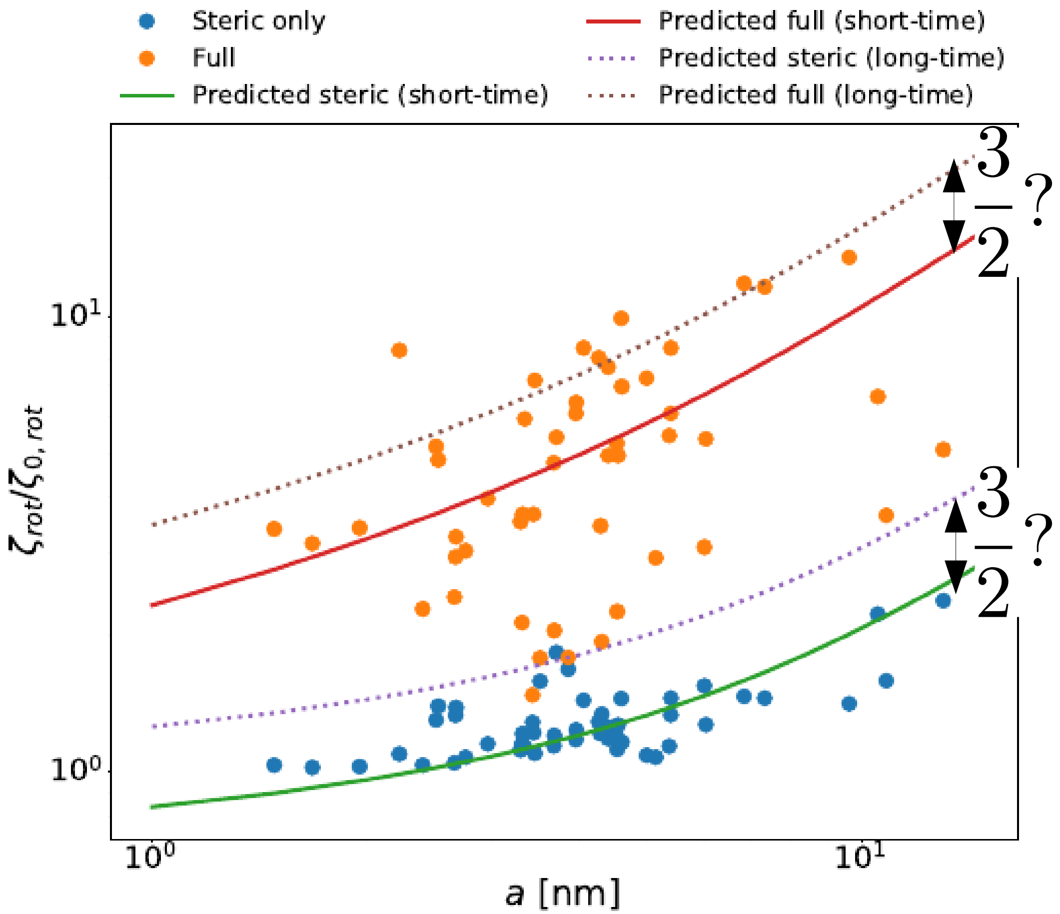
universality

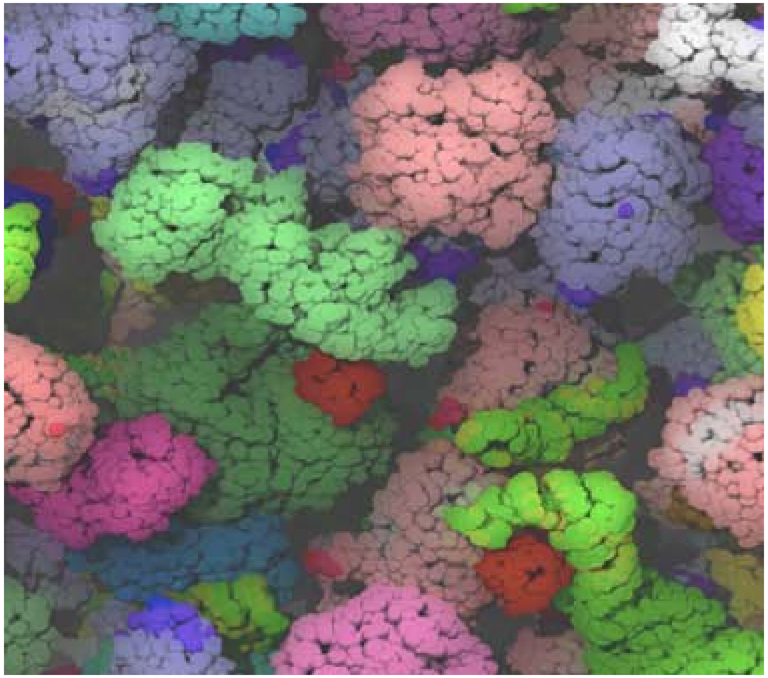
Verification of our phenomenological Stokes' law

$$\frac{1}{\zeta_{rot}(a)} = -\frac{3}{4a}\frac{d}{da}\frac{1}{\zeta(a)}$$

Experiments: our work: ongoing, literature: we didn't find
Numerical simulations – similar situation, but...

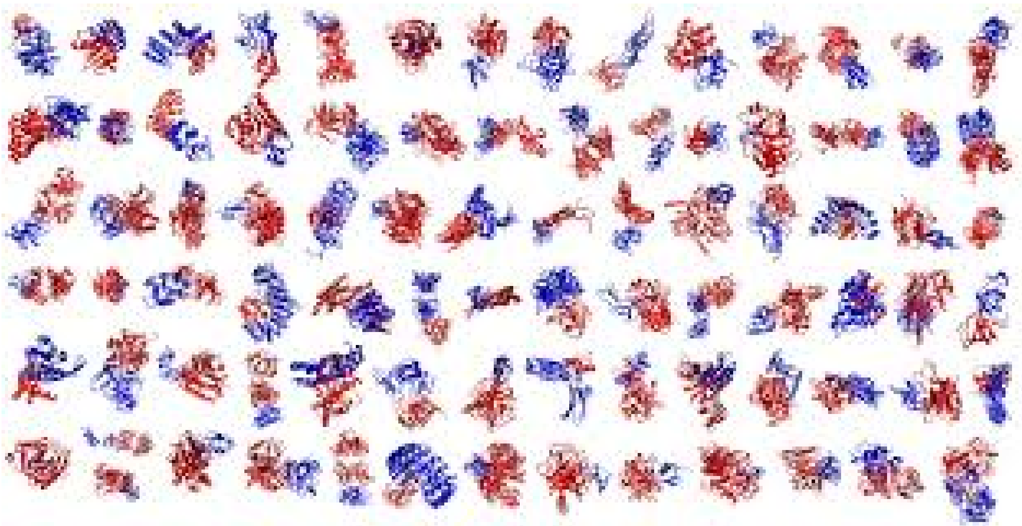
Sean R. McGuffee, Adrian H. Elcock (2010):

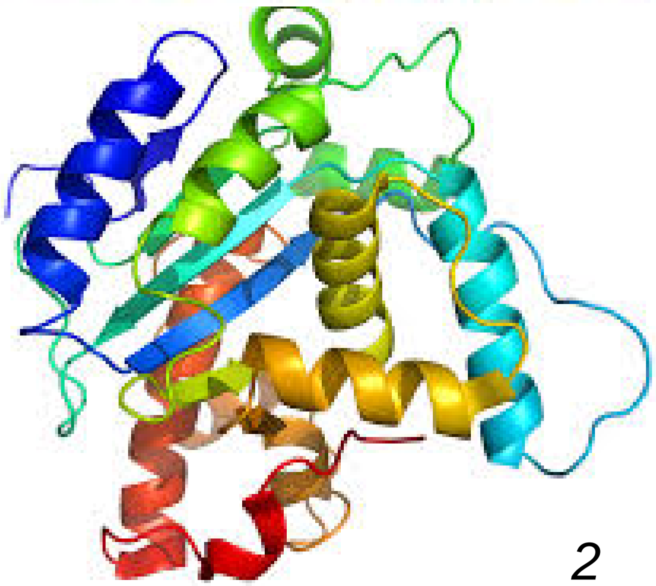




Outlook

Simplified description of life processes in cells...





in $\eta(k)$

2

Inertial microfluidics: in complex liquids ‘Re’ can be large for small particles but small for big particles